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Inclusive heavy hadron decays and light-cone dynamics

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Abstract

The governing role of light-cone dynamics in inclusive heavy hadron decay processes is demonstrated. Nonperturbative QCD effects on the processes can be systematically calculated using light-cone expansion and heavy quark effective theory. The applications of the light-cone approach to studying electroweak and strong interactions and hadron structure with semileptonic and radiative decays of beauty hadrons are briefly reviewed.

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INCLUSIVE HEAVY HADRON DECAYS AND LIGHT-CONE DYNAMICS

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The governing role of light-cone dynamics in inclusive heavy hadron decay processes is demonstrated. Nonperturbative QCD effects on the processes can be systematically calculated using light-cone expansion and heavy quark effective theory. The applications of the light-cone approach to studying electroweak and strong interactions and hadron structure with semileptonic and radiative decays of beauty hadrons are briefly reviewed.

1 Introduction

The beauty hadron is the heaviest hadron containing a single heavy quark but lives for a long time, while the top quark is too heavy to build hadrons and has a very short lifetime. This confers a special role to beauty hadrons in heavy hadron physics: The longevity of the beauty hadron makes it phenomenologically interesting, and the heaviness of the beauty hadron ensures theoretical reliability.

Because of the long lifetime of the beauty hadron, many interesting phenomena have chance to be observed. We discuss here semileptonic or radiative inclusive decays of B mesons. The discussion can be easily extended to inclusive decays of other heavy hadrons. Semileptonic or radiative inclusive decays of B mesons are interesting because these decays are our main tool to determine the fundamental Cabibbo-Kobayashi-Maskawa (CKM) matrix elements, to probe hadron structure and strong interactions, to test the standard model, and to search for new physics beyond the standard model.

The principal topic in the theoretical description of inclusive decays of B mesons is the calculation of nonperturbative QCD contributions. There are essentially two components of nonperturbative QCD effects: one on dynamics — it is responsible for the confinement of quarks inside a hadron, the other on kinematics — it transforms parton kinematics into hadron kinematics. The past decade saw great progress in the QCD treatment of inclusive decays of heavy hadrons, including the construction of the heavy quark effective theory [1] and the developments of the heavy quark expansion approach [2] and the light-cone approach [3,4,5,6,7]. In this talk I describe the light-cone approach and present a brief review of some of its applications. A comparison of the

light-cone approach and the heavy quark expansion approach can be found in Refs. [8,9].

The light-cone approach uses the methods of light-cone expansion and heavy quark effective theory to address both dynamic and kinematic effects of nonperturbative QCD. This QCD approach has led to an improvement over the naive parton model [10] for inclusive B decays, so that model-independent predictions and a control over theoretical uncertainties become possible.

2 Light-Cone Expansion

All nonperturbative QCD physics for inclusive semileptonic B decays are incorporated in the hadronic tensor:

$$W_{\mu\nu} = -\frac{1}{2\pi}\int d^4y\; e^{iq\cdot y}\langle B\left|[j_\mu(y), j_\nu^\dagger(0)]\right|B\rangle, \tag{1}$$

where q stands for the momentum of the virtual W boson. In general, the hadronic tensor can be expressed in terms of five scalar functions $W_{1-5}(\nu,\, q^2)$ of the two independent Lorentz invariants, $\nu = q\cdot P/M_B$ and q^2 (M_B and P denote the mass and momentum of the B meson, respectively),

$$W_{\mu\nu} = -g_{\mu\nu}W_1 + \frac{P_\mu P_\nu}{M_B^2}W_2 - i\varepsilon_{\mu\nu\alpha\beta}\frac{P^\alpha q^\beta}{M_B^2}W_3 + \frac{q_\mu q_\nu}{M_B^2}W_4 + \frac{P_\mu q_\nu + q_\mu P_\nu}{M_B^2}W_5\,. \tag{2}$$

The theoretical task is to calculate the five structure functions.

Let's look into the expression (1) for the hadronic tensor. The current commutator in Eq. (1) is causal, $[j_\mu(y), j_\nu^\dagger(0)] = 0$ for $y^2 < 0$. Moreover, $e^{iq\cdot y}$ in Eq. (1) oscillates rapidly, averaging out contributions to the integral except in the domain $y^2 \le 1/q^2$. For inclusive semileptonic B decays, the momentum transfer squared lies in the range $M_\ell^2 \le q^2 \le (M_B - M_{X_{\rm min}})^2$, where M_ℓ is the charged lepton mass and $M_{X_{\rm min}}$ is the minimal hadronic invariant mass in the final state, which is the D meson (pion) mass for the $b\to c$ ($b\to u$) transition. Because of the heaviness of the B meson, the decays occur mostly at large momentum transfer. Therefore, the combined consideration of causality, less rapid oscillations and kinematics leads us to realize that the hadronic tensor is dominated by the space-time near the light cone $y^2 \to 0$.

Another argument for the light-cone dominance is as follows. One can express the commutator of two currents in terms of the bilocal operator:

$$\left[j_\mu(y), j_\nu^\dagger(0)\right] = 2(S_{\mu\alpha\nu\beta} - i\varepsilon_{\mu\alpha\nu\beta})\,[\partial^\alpha\Delta_q(y)]\,\bar b(0)\gamma^\beta \mathcal{P}exp[ig_s\int_y^0 dz^\mu A_\mu(z)]b(y)\,, \tag{3}$$

where $S_{\mu\alpha\nu\beta} = g_{\mu\alpha}g_{\nu\beta} + g_{\mu\beta}g_{\nu\alpha} - g_{\mu\nu}g_{\alpha\beta}$ and $\Delta_q(y)$ is the Pauli-Jordan function. $\mathcal{P}$ denotes path ordering. The coefficient function of the bilocal operator in Eq. (3) is finite except on the light cone $y^2 = 0$ where it is singular. Therefore, the light-cone singularity also implies that the dominant contribution to the hadronic tensor comes from the space-time separations in the neighborhood of the light cone.

The light-cone dominance justifies the light-cone expansion in matrix elements of increasing twist. The light-cone expansion provides a formal and powerful way of organizing the nonperturbative QCD effects and singling out the leading term. The leading nonperturbative QCD contribution to inclusive semileptonic decays of B mesons resides in the distribution function [7]

$$f(\xi) = \frac{1}{4\pi}\int \frac{d(y\cdot P)}{y\cdot P} e^{i\xi y\cdot P}\langle B|\bar{b}(0)\not{y}\mathcal{P}exp[ig_s\int_y^0 dz^\mu A_\mu(z)]b(y)|B\rangle|_{y^2=0}\,. \tag{4}$$

The five structure functions, *a priori* independent, are then related to a single distribution function in leading twist approximation [3,4,5,6,7]:

$$W_1 = 2[f(\xi_+) + f(\xi_-)]\,, \tag{5}$$

$$W_2 = \frac{8}{\xi_+ - \xi_-}[\xi_+ f(\xi_+) - \xi_- f(\xi_-)]\,, \tag{6}$$

$$W_3 = -\frac{4}{\xi_+ - \xi_-}[f(\xi_+) - f(\xi_-)]\,, \tag{7}$$

$$W_4 = 0\,, \tag{8}$$

$$W_5 = W_3\,, \tag{9}$$

where $\xi_\pm = (\nu \pm \sqrt{\nu^2 - q^2 + m_q^2})/M_B$.

It has been shown [7] that inclusive radiative decays of B mesons $B \to X_s\gamma$ are also dominated by light-cone dynamics. It turns out that the distribution function is universal: The same distribution function encodes the leading nonperturbative QCD contribution to inclusive radiative decays of B mesons. The universality originates from the fact that the primary object of analysis in long-distance effects is the same bilocal operator matrix element dictated by light-cone dynamics. In leading twist approximation, the photon energy spectrum is then given by [7]

$$\frac{d\Gamma(B \to X_s\gamma)}{dE_\gamma} = \frac{G_F^2\alpha m_b^2}{2\pi^4 M_B}|V_{tb}V_{ts}^*|^2|C_7^{(0)}(M_W)|^2 E_\gamma^3 f\left(\frac{2E_\gamma}{M_B}\right). \tag{10}$$

3 Properties of the Distribution Function

The b-quark distribution function $f(\xi)$ for the B meson is a key object. Like the well-known parton distribution functions for the nucleon in deep inelastic lepton-nucleon scattering, the knowledge of the b-quark distribution function would help us greatly in understanding the nature of confinement and the structure of the B meson. I will survey our present knowledge of the b-quark distribution function and look forward to likely advances in the future.

The distribution function is gauge invariant. It obeys positivity and the support of it is $0 \leq \xi \leq 1$. The distribution function is exactly normalized to unity

$$\int_0^1 d\xi \, f(\xi) = 1. \tag{11}$$

This normalization does not get renormalized as a consequence of b-flavored quantum number conservation. The distribution function contains the free quark decay as a limiting case with $f(\xi) = \delta(\xi - m_b/M_B)$. The distribution function $f(\xi)$ has a simple physical interpretation: It is the probability of finding a b-quark with momentum ξP inside the B meson with momentum P.

In addition, the mean μ and the variance σ^2 of the distribution function were deduced [7] using the techniques of the operator product expansion and the heavy quark effective theory (HQET):

$$\mu \equiv \int_0^1 d\xi \, \xi f(\xi) = \frac{m_b}{M_B}\left(1 + \frac{5E_b}{3}\right), \tag{12}$$

$$\sigma^2 \equiv \int_0^1 d\xi \, (\xi - \mu)^2 f(\xi) = \left(\frac{m_b}{M_B}\right)^2 \left[\frac{2K_b}{3} - \left(\frac{5E_b}{3}\right)^2\right], \tag{13}$$

where $E_b = K_b + G_b$ and K_b and G_b are the dimensionless HQET parameters of order $(\Lambda_{\rm QCD}/m_b)^2$, which are often referred to by the alternate names $\lambda_1 = -2m_b^2 K_b$ and $\lambda_2 = -2m_b^2 G_b/3$. The parameter λ_2 can be extracted from the $B^* - B$ mass splitting: $\lambda_2 = (M_{B^*}^2 - M_B^2)/4 = 0.12 \text{ GeV}^2$. The parameter λ_1 suffers from large uncertainty.

The mean value and variance of the distribution function characterize the location of the "center of mass" of the distribution function and the square of its width, respectively. They specify the primary shape of the distribution function. From Eqs. (12) and (13) we know that the distribution function is sharply peaked around $\xi = \mu \approx m_b/M_B$ close to 1 and its width of order $\Lambda_{\rm QCD}/M_B$ is narrow, suggesting that the distribution function is close to the

delta function form in the free quark limit. Note that the b-quark distribution function has different forms for different beauty hadrons.[6]

Nonperturbative QCD methods such as lattice gauge theory, light-cone field theory, and QCD sum rules could help determine further the form of the distribution function.

The b-quark distribution function can also be extracted directly from experiment. The $B \to X_s\gamma$ photon energy spectrum [7] and the $B \to X_q\ell\nu$ spectra $d\Gamma/d\xi_+$ [11,8] share a common feature — namely they offer the intrinsically most sensitive probe of long-distance strong interactions because these spectra correspond to a discrete line solely on kinematic grounds in the absence of gluon bremsstrahlung and long-distance strong interactions. Indeed, our calculation based on the light-cone expansion shows that they are explicitly proportional to the nonperturbative distribution function. Therefore, the shapes of these spectra directly reflect the inner long-distance dynamics of the reactions and measurements of these spectra are idealy suited for direct extraction of the distribution function from experiment.

The universality of the distribution function implies great predictive power: Once the distribution function is measured from one process, it can be used to make predictions in all other processes in a model-independent manner.

4 Applications

I have described the QCD approach to inclusive decays of heavy hadrons from light-cone expansion and heavy quark effective theory. Because of the large mass of the decaying hadron, light-cone dynamics plays a governing role in inclusive decays of heavy hadrons. The light-cone expansion allows a rigorous and systematic ordering of nonperturbative QCD effects, and the identification of the leading term — the b-quark distribution function. Another large scale set by the b-quark mass allows the construction of the heavy quark effective theory, providing another complementary framework for organizing and parametrizing nonperturbative QCD effects. The additional properties of the distribution function have been learned exploiting the heavy quark effective theory.

This approach has been applied to calculate decay rates and distributions in semileptonic or radiative inclusive decays of B mesons. Model-independent predictions have been made from the known normalization of the distribution function or the cancellation of the distribution function in the ratio of the decay rates. Some calculations of decay rates and distributions involve the modelling of the distribution function, since the form of it has as yet not been

completely determined. However, for sufficiently inclusive quantities such as the total semileptonic decay rates, the results are nearly model-independent, since they are essentially only sensitive to the mean value and variance of the distribution function, which are known from the heavy quark effective theory. Direct extraction of the distribution function from precision measurements would eliminate the model dependence.

A crucial observation is that both dynamic and kinematic effects of nonperturbative QCD must be taken into account.[4,12,9] The latter results in the extension of phase space from the quark level determined by the b-quark mass to the hadron level determined by the B meson mass, thereby increasing decay rates. The calculations in the light-cone approach are able to include both dynamic and kinematic effects of nonperturbative QCD, and have shown that the net effect of nonperturbative QCD enhances the total semileptonic rates for $B \to X_c \ell \nu$ [4] and $B \to X_u \ell \nu$ [12], contrary to the results [13] obtained in the heavy quark expansion approach. The heavy quark expansion approach [2] assumes quark-hadron duality, and cannot account for the rate due to the extension of phase space from the quark level to the hadron level. The light-cone approach provides a way around problems associated with the assumption of quark-hadron duality. The rate calculations in the light-cone approach have quantitatively shown [4,12] the importance of the inclusion of kinematic effects of nonperturbative QCD.

The interplay between nonperturbative and perturbative QCD effects has been accounted for in the light-cone approach, since confinement implies that free quarks are not asymptotic states of the theory and the separation of perturbative and nonperturbative effects cannot be done in a clear-cut way.

The following are some of the results.

(1) A new method for precise and model-independent determination of $|V_{ub}|$ has been proposed.[11,8] For inclusive charmless semileptonic decays of B mesons $B \to X_u \ell \nu$, the light-cone expansion and b-flavored quantum number conservation lead to the sum rule [11]

$$S \equiv \int_0^1 d\xi_u \, \frac{1}{\xi_u^5} \frac{d\Gamma}{d\xi_u}(B \to X_u \ell \nu) = |V_{ub}|^2 \frac{G_F^2 M_B^5}{192\pi^3} \,, \tag{14}$$

with the kinematic variable $\xi_u = (q^0 + |\vec{q}|)/M_B$ in the B-meson rest frame. This sum rule receives no perturbative QCD correction, and avoids the dominant hadronic uncertainty. The sum rule (14) thus establishes a clean relationship between $|V_{ub}|$ and the observable S, allowing precise determination of $|V_{ub}|$. This determination of $|V_{ub}|$ is also model-independent in the sense that the sum rule is independent of phenomenological models. Moreover, this method is not only exceptionally clean theoretically, but also very effi-

cient experimentally in background suppression. Applying the kinematic cut $q^0 > M_B - M_D$ or $M_X < M_D$, which leads to $\xi_u > 1 - M_D/M_B$, one can discriminate between $b \to u$ signal and $b \to c$ background. The ξ_u spectrum is unique. About 80% of the spectrum satisfy $\xi_u > 1 - M_D/M_B$.

(2) $|V_{ub}|$ has been determined [12] from the measured inclusive charmless semileptonic branching ratio of beauty hadrons. The determination of $|V_{ub}|$ from the measurement of the inclusive charmless semileptonic branching ratio in conjunction with the calculation of the total charmless semileptonic rate has larger theoretical uncertainty than the proposed determination of $|V_{ub}|$ from the measurement of the observable S in conjunction with the sum rule (14), since the calculations of perturbative and nonperturbative QCD corrections to the total charmless semileptonic rate suffer from much larger theoretical uncertainty [12] than the sum rule (14), not to mention the fundamental uncertainty due to the assumption of quark-hadron duality if the total semileptonic rate is calculated in the heavy quark expansion approach.

(3) The $B \to X_u \ell \nu$ ($\ell = e$ or μ) charged lepton energy spectrum [5], hadronic invariant mass spectrum [14], and lepton pair spectrum [8] have been analyzed in detail, resulting in the state-of-the-art descriptions of these spectra. It has been shown that the interplay between perturbative and nonperturbative QCD eliminates the singularities of the parton-level perturbative spectra.

(4) $|V_{cb}|$ has been determined [4] from the measured inclusive semileptonic branching ratio of beauty hadrons in conjunction with the calculation of the total semileptonic rate. The charged lepton energy spectrum in inclusive semileptonic decays of B mesons $B \to X_c \ell \nu$ has been analyzed [5] in detail, resulting in the state-of-the-art description of the spectrum. The calculated spectrum was found [5] to be in agreement with the experimental data, providing an experimental test of the validity of the light-cone approach.

(5) The total semileptonic decay rate of the Λ_b baryon has been calculated.[6] The calculated semileptonic branching ratio for Λ_b is consistent with the measurements.

(6) Nonperturbative QCD effects on the $B \to X_s \gamma$ photon energy spectrum have been calculated.[7] The theoretically clean methods for the determinations of $|V_{ts}|$, $|V_{ts}/V_{ub}|$ and $|V_{ts}/V_{cb}|$ have been suggested.[7]

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